EE363 Homework 1

Spring 2026

2.20. State equations for a linear mechanical system. The equations of motion of a lumped mechanical system undergoing small motions can be expressed as

$$M\ddot{q} + D\dot{q} + Kq = f$$

where $q(t) \in \mathbb{R}^k$ is the vector of deflections, M, D, and K are the *mass*, *damping*, and *stiffness* matrices, respectively, and $f(t) \in \mathbb{R}^k$ is the vector of externally applied forces. Assuming M is invertible, write linear system equations for the mechanical system, with state

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix},$$

input $u = f$, and output $y = q$.

2.70. Matrix representation of linear systems. Consider the (discrete-time) linear dynamical system

$$x(t+1) = A(t)x(t) + B(t)u(t), \quad y(t) = C(t)x(t) + D(t)u(t).$$

Find a matrix G such that

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N) \end{bmatrix} = G \begin{bmatrix} x(0) \\ u(0) \\ \vdots \\ u(N) \end{bmatrix}.$$

The matrix G shows how the output at $t = 0, \ldots, N$ depends on the initial state $x(0)$ and the sequence of inputs $u(0), \ldots, u(N)$.

9.1460. Linear dynamical system with constant input. We consider the system $\dot{x} = Ax + b$, with $x(t) \in \mathbb{R}^n$. A vector x_e is an equilibrium point if $0 = Ax_e + b$. (This means that the constant trajectory $x(t) = x_e$ is a solution of $\dot{x} = Ax + b$.)

a) When is there an equilibrium point?

b) When are there multiple equilibrium points?

c) When is there a unique equilibrium point?

d) Now suppose that x_e is an equilibrium point. Define $z(t) = x(t) - x_e$. Show that $\dot{z} = Az$. From this, give a general formula for $x(t)$ (involving x_e, $\exp(tA)$, $x(0)$).

e) Show that if all eigenvalues of A have negative real part, then there is exactly one equilibrium point x_e, and for any trajectory $x(t)$, we have $x(t) \to x_e$ as $t \to \infty$.

11.1740. Another formula for the matrix exponential. You might remember that for any complex number $a \in \mathbb{C}$, $e^a = \lim_{k\to\infty}(1 + a/k)^k$. You will establish the matrix analog: for any $A \in \mathbb{R}^{n\times n}$,

$$e^A = \lim_{k\to\infty} (I + A/k)^k.$$

To simplify things, you can assume A is diagonalizable. *Hint:* diagonalize.

12.1920. Jordan form of a block matrix. We consider the block 2×2 matrix

$$C = \begin{bmatrix} A & I \\ 0 & A \end{bmatrix}.$$

Here $A \in \mathbb{R}^{n \times n}$, and is diagonalizable, with real, distinct eigenvalues $\lambda_1, \ldots, \lambda_n$. We'll let $v_1, \ldots, v_n$ denote (independent) eigenvectors of A associated with $\lambda_1, \ldots, \lambda_n$.

a) Find the Jordan form J of C. Be sure to explicitly describe its block sizes.

b) Find a matrix T such that $J = T^{-1}CT$.