EE363 Homework 2

Spring 2026

11.1780. Companion matrices. A matrix A of the form

$$A = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

is said to be a (top) *companion matrix*. There can be four forms of companion matrices depending on whether the a_i's occur in the first or last row, or first or last column. These are referred to as top-, bottom-, left-, or right-companion matrices. Let $\dot{x} = Ax$ where A is top-campanion.

a) Draw a block diagram for the system $\dot{x} = Ax$.

b) Find the characteristic polynomial of the system using the block diagram and show that A is nonsingular if and only if $a_n \neq 0$.

c) Show that if A is nonsingular, then A^{-1} is a bottom-companion matrix with last row $-[1\ a_1\ \cdots\ a_{n-1}]/a_n$.

d) Find the eigenvector of A associated with the eigenvalue λ.

e) Suppose that A has distinct eigenvalues $\lambda_1, \ldots, \lambda_n$. Find T such that $T^{-1}AT$ is diagonal.

12.1910. Asymptotically periodic trajectories. We say that $x : \mathbb{R}_+ \to \mathbb{R}^n$ is *asymptotically* T-*periodic* if $\|x(t+T) - x(t)\|$ converges to 0 as $t \to \infty$. (We assume $T > 0$ is fixed.) Now consider the (time-invariant) linear dynamical system $\dot{x} = Ax$, where $A \in \mathbb{R}^{n \times n}$. Describe the precise conditions on A under which *all* trajectories of $\dot{x} = Ax$ are asymptotically T-periodic. Give your answer in terms of the Jordan form of A. (The period T can appear in your answer.) Make sure your answer works for 'silly' cases like $A = 0$ (for which all trajectories are constant, hence asymptotically T-periodic), or stable systems (for which all trajectories converge to 0, hence are asymptotically T-periodic). Mark your answer clearly, to isolate it from any (brief) discussion or explanation. You do not need to formally prove your answer; a brief explanation will suffice.

12.1930. Properties of trajectories. For each of the following statements, give the exact (necessary and sufficient) conditions on $A \in \mathbb{R}^{n \times n}$ under which the statement holds.

a) Every trajectory of $\dot{x} = Ax$ converges as $t \to \infty$. This means that, for any $x(0)$, $x(t)$ converges to some value, which need not be zero (and can depend on $x(0)$ and A).

b) Every trajectory of $\dot{x} = Ax$ is bounded. This means that, for any $x(0)$, there is an M (that can depend on $x(0)$ and A) for which $\|x(t)\| \leq M$ for all $t \geq 0$.

Your answers can refer to any concepts used in the course (eigenvalues, singular values, Jordan form, least-squares, range, nullspace, ...). We will deduct points from answers that are technically correct, but more complicated than they need to be. *You may not make any assumptions about A* (*e.g.*, that it is nonsingular, diagonalizable, etc.).

Please give only your final answer; we do not want any justification or discussion. Your answers should have a form similar to "The property in part (a) occurs if and only if all singular values of A are less than one, and A has no real eigenvalues". (This is *not* the correct answer; it is only as an example of what your answer should look like.)

13.2000. Inverse of a linear system. Suppose $H(s) = C(sI - A)^{-1}B + D$, where D is square and invertible. You will find a linear system with transfer function $H(s)^{-1}$.

a) Start with $\dot{x} = Ax + Bu$, $y = Cx + Du$, and solve for $\dot{x}$ and u in terms of x and y. Your answer will have the form: $\dot{x} = Ex + Fy$, $u = Gx + Hy$. Interpret the result as a linear system with state x, input y, and output u.

b) Verify that

$$(G(sI - E)^{-1}F + H)(C(sI - A)^{-1}B + D) = I.$$

Hint: use the following "resolvent identity:"

$$(sI - X)^{-1} - (sI - Y)^{-1} = (sI - X)^{-1}(X - Y)(sI - Y)^{-1}$$

which can be verified by multiplying by $sI - X$ on the left and $sI - Y$ on the right.