

Example: Input Design

EE363
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Examples (ideas only, no details)

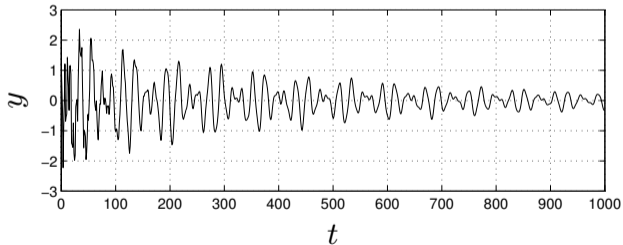
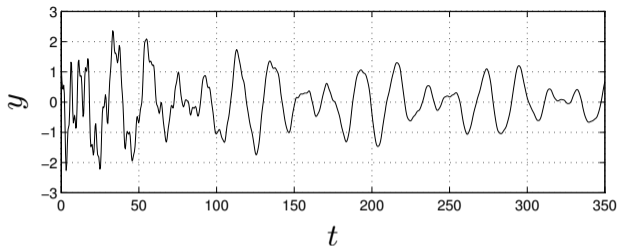
- ▶ let's consider a specific system

$$\dot{x} = Ax, \quad y = Cx$$

with $x(t) \in \mathbb{R}^{16}$, $y(t) \in \mathbb{R}$ (a '16-state single-output system')

- ▶ model of a lightly damped mechanical system, but it doesn't matter

Typical output

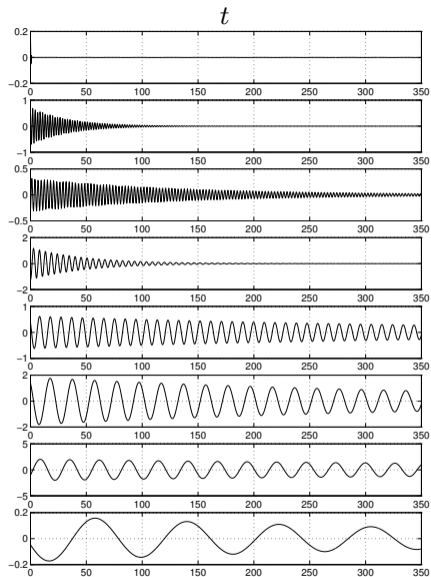


- ▶ output waveform is very complicated; looks almost random and unpredictable

Modal components

we'll see that such a solution can be decomposed into much simpler (modal) components

(idea probably familiar)



Input design

add two inputs, two outputs to system:

$$\dot{x} = Ax + Bu, \quad y = Cx, \quad x(0) = 0$$

where $B \in \mathbb{R}^{16 \times 2}$, $C \in \mathbb{R}^{2 \times 16}$ (same A as before)

problem: find appropriate $u : \mathbb{R}_+ \rightarrow \mathbb{R}^2$ so that $y(t) \rightarrow y_{\text{des}} = (1, -2)$

simple approach: consider static conditions (u , x , y constant):

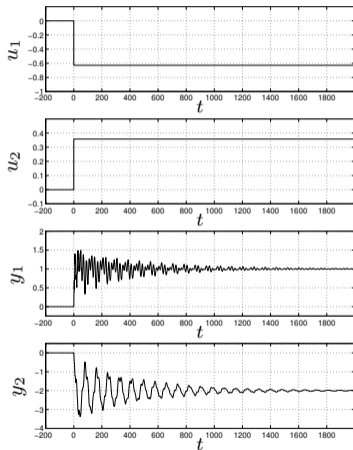
$$\dot{x} = 0 = Ax + Bu_{\text{static}}, \quad y = y_{\text{des}} = Cx$$

solve for u to get:

$$u_{\text{static}} = (-CA^{-1}B)^{-1} y_{\text{des}} = \begin{bmatrix} -0.63 \\ 0.36 \end{bmatrix}$$

Simple approach

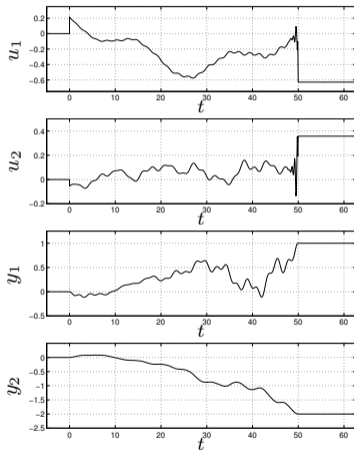
let's apply $u = u_{\text{static}}$ and just wait for things to settle:



...takes about 1500 sec for $y(t)$ to converge to y_{des}

Faster convergence

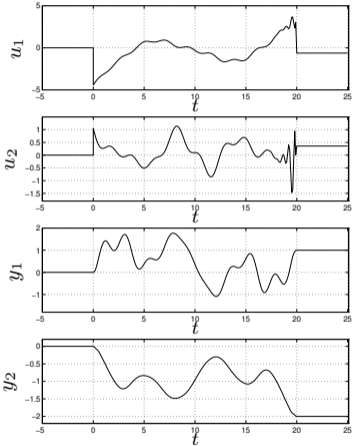
using very clever input waveforms (EE363) we can do much better, *e.g.*



... here y converges *exactly* in 50 sec

Still faster

in fact by using larger inputs we do still better, *e.g.*



... here we have (exact) convergence in 20 sec

in this course we'll study

- ▶ how to synthesize or design such inputs
- ▶ the tradeoff between size of u and convergence time